Sesión Especial 6

Red de Geometría Algebraica y Singularidades

Organizadores:

- Enrique Artal Bartolo (Universidad de Zaragoza)
- José Ignacio Cogolludo Agustín (Universidad de Zaragoza)
- Ilya Smirnov (Basque Center for Applied Mathematics)

Descripción:
La Red de Geometría Algebraica y Singularidades está reconocida por el MINECO (MTM2016-81735-REDT, RED2018-102583-T, RED2022-134105-T) desde el año 2016, y está formada por más de 120 investigadores integrados en 27 proyectos de investigación financiados por el MINECO y organizados en 13 nodos. La sesión tiene un triple objetivo:

- Presentar los recientes avances en el ámbito de la Geometría Algebraica y las Singularidades.
- Apoyar las actividades de la RSME en cuanto a difusión y visibilidad científica de las matemáticas en España.
- Hacer visible la Red y su investigación en el contexto nacional. La información completa de la Red se puede consultar en su página web: http://www.mat.ucm.es/~rgas/.

Programa

LUNES, 22 de enero:

16:00 – 16:30	Francisco Monserrat (U. Politécnica de Valencia) *On the finite generation of the effective cone and the Cox ring of a rational surface*
16:30 – 17:00	Marta Aldasoro (BCAM) *Some consequences of the μ-constant condition for families of surfaces*
17:00 – 17:30	Moisés Herradón Cueto (UAM) *Hodge Theory of abelian covers of algebraic varieties*
17:30 – 18:00	Baldur Sigurðsson (U. Complutense de Madrid) *The total spine and invariant Milnor fibration at radius zero for plane curve singularities*

MARTES, 23 de enero:

11:30 – 12:00	Josep Àlvarez Montaner (U. Politècnica de Catalunya) *Operadores diferenciales en anillos de invariantes de grupos finitos*
12:00 – 12:30	Pablo Hernández (U. de Salamanca) *Robustez de agujeros en complejos simpliciales finitos*
12:30 – 13:00	Juan Carlos Naranjo (U. de Barcelona) *Sophie Germain coverings of curves of genus* 2
13:00 – 13:30	María Cruz Fernández Fernández (U. de Sevilla) *Sobre la multisumabilidad de las series hipergeométricas*
16:00 – 16:30	Guillermo Peñafort Sanchís (U. de València) *La conjetura de Mond*
16:30 – 17:00	Luis José Santana Sánchez (U. de Valladolid) *Duality of divisors and curves on Mori dream spaces*
17:00 – 17:30	Carlos Jesús Moreno Ávila (U. Jaume I) *Visualizing the valuative Nagata conjecture: A geometric approach*
17:30 – 18:00	Eslam Badr (American University in Cairo) *On automorphism groups and twisting theory for smooth plane curves*

On the finite generation of the effective cone and the Cox ring of a rational surface

Francisco Monserrat, C. Galindo and C. J. Moreno Ávila

Universidad Politécnica de Valencia

framonde@mat.upv.es

Abstract: Let X be a rational surface obtained from a Hirzebruch surface by a sequence of blowups centered at closed points. We will show conditions which imply, on the one hand, that the effective cone of X is polyhedral and minimally generated and, on the other hand, the finite generation of the Cox ring of X. Moreover, we provide a set of generators of the nef cone of X in these cases. The talk is based on joint work with C. Galindo and C. J. Moreno Ávila.

Some consequences of the μ-constant condition for families of surfaces.

Marta Aldasoro, Javier Fernández de Bobadilla

Basque Center for Applied Mathematics

maldasoro@bcamath.org

Abstract: We work with a 1-parameter family $X \to \Delta$ of isolated hypersurface singularities of fibre dimension 2 with constant Milnor number. Let $W \to X$ be a semistable resolution of singularities, obtained replacing X by a suitable base change. We show that the second cohomology groups of the exceptional divisors lying in the central fibre W_0 satisfy the vanishing $h^{2,0} = 0$. Furthermore, we prove that the dual complex associated to the mentioned central fibre has the rational homology of a point. These two results allow us to show that if we make a compactification of our family, then the first cohomology group of the compactified central fibre coincides with that of the compactification of the strict transform. To achieve them, we make use of the Steenbrink spectral sequence degenerating to the cohomology of a generic fibre of the resolution, endowed with the limit mixed Hodge structure.

Hodge Theory of abelian covers of algebraic varieties

Moisés Herradón Cueto, Eva Elduque

Universidad Autónoma de Madrid

moises.herradon@uam.es

Abstract: Let $f : U \to \mathbb{C}^*$ be an algebraic map from a smooth complex connected algebraic variety U to the punctured complex line $\mathbb{C}^*$. Using f to pull back the exponential map $C \to \mathbb{C}^*$, one obtains an infinite cyclic cover U^f of the variety U, together with a $\mathbb{Z}$-action coming from adding $2\pi i$ in $\mathbb{C}$. The homology groups of this infinite cyclic cover, with their $\mathbb{Z}$-actions, are the family of Alexander modules associated to f. In previous work jointly with Eva Elduque, Christian Geske, Laurențiu Maxim and Botong Wang, we constructed a mixed Hodge structure on the torsion part of these Alexander modules. In this talk, we will talk about work in progress aimed at generalizing this theory to abelian covering spaces of algebraic varieties which arise in an algebraic way, i.e. from maps $f : U \to G$, where G is a semiabelian variety. This is joint work with Eva Elduque.

The total spine and invariant Milnor fibration at radius zero for plane curve singularities

Baldur Sigurðsson, Pablo Portilla Cuadrado

Universidad Complutense de Madrid

baldursi@ucm.es

Abstract: Joint work with Pablo Portilla Cuadrado. We consider a plane curve singularity defined by a holomorphic function germ f in two complex variables. The total spine is the union of trajectories of the negative gradient flow of $|f|$ which converge to the origin. We prove that for a generic metric, this set is the disjoint union of finitely many submanifolds. The invariant Milnor fibration at radius zero is obtained by collapsing a finite number of embedded disks in the Milnor fibration at radius zero. Its intersection with the strict transform of the total spine is a spine for this fibration, and thus carries algebro-topological invariants of the Milnor fibration. In ongoing work, we apply this situation to study the monodromy, and in particular, the variation map.

Operadores diferenciales en anillos de invariantes de grupos finitos

Josep Àlvarez Montaner

Universitat Politècnica de Catalunya

josep.alvarez@upc.es

Resumen: En esta charla vamos a ver que ciertos aspectos de la teoría de operadores diferenciales sobre anillos de polinomios se pueden extender al caso de anillos de invariantes de grupos finitos. En particular podemos definir la noción de módulos holónomos y estudiar la cohomología de de Rham de estos módulos.

Robustez de agujeros en complejos simpliciales finitos

Pablo Hernández

Universidad de Salamanca

pablohg.eka@usal.es

Resumen: La interpretación de los complejos simpliciales como variedades proyectivas y la teoría de Stanley-Reisner sugiere el uso de herramientas y técnicas de geometría/topología algebraica en el estudio de la combinatoria presente en la teoría de redes. En particular, los números de Betti son importantes invariantes topológicos que proporcionan información sobre las componentes conexas y agujeros del complejo simplicial que modeliza la red. En esta charla veremos cómo por un lado el estudio de los coesqueletos y por otro de la teoría de haces sobre espacios topológicos finitos nos permite definir nuevos conjuntos de invariantes asociados a la categoría de complejos simpliciales finitos, los cuales contienen como caso particular a los números de Betti. Finalmente mostraremos cómo la functorialidad de las construcciones permite interpretar estas generalizaciones de los números de Betti como una medida de la robustez de las componentes conexas y agujeros de un complejo simplicial.

Sophie Germain coverings of curves of genus 2

Juan Carlos Naranjo, A. Ortega (Humboldt U.) and I. Spelta (CRM)

Universitat de Barcelona

jcnaranjo@ub.edu

Abstract: We consider unramified cyclic coverings of odd degree d of curves of genus 2. By a result of Lange and Ortega, it is known that the corresponding Prym map $\mathcal{P}_d$ has degree 10 for $d = 7$, and Albano and Pirola proved that the generic fibers of $\mathcal{P}_3$ and $\mathcal{P}_5$ are positive dimensional. Moreover, Agostini proved that $\mathcal{P}_d$ is generically finite for $d \geq 7$. In this talk I will report on a proof of the generic injectivity for $\mathcal{P}_d$ for $d = 2k + 1$ prime such that k is also prime. It is conjectured that there are infinitely many pairs of prime numbers of the form $(k, 2k+1)$. These are called Sophie Germain prime numbers.
Our method is based on the study of the isogeny type of the Prym variety and the computation of the theta dual variety of some distinguished curves. This is a joint work with A. Ortega (Humboldt U.) and I. Spelta (CRM).

Sobre la multisumabilidad de las series hipergeométricas

María Cruz Fernández Fernández, Francisco Jesús Castro Jiménez y Saiei-Jaeyeong Matsubara-Heo

Universidad de Sevilla

mcferfer@us.es

Resumen: Los D-módulos hipergeométricos fueron introducidos por Gelfand, Kapranov y Zelevinsky a finales de los 80 y están asociados a un ideal tórico y a un vector de parámetros complejos. Es bien conocido que estos D-módulos son regulares holónomos exactamente cuando el ideal tórico es homogéneo respecto a la graduación estándar. Una serie hipergeométrica es una solución formal de un D-módulo hipergeométrico irregular. En un trabajo previo conjunto con Francisco Jesús Castro Jiménez, Tatsuya Koike y Nobuki Takayama, probamos que ciertas series hipergeométricas son sumables Borel. En esta charla presentamos un trabajo en progreso conjunto con Francisco Jesús Castro Jiménez y Saiei-Jaeyeong Matsubara-Heo. En particular, mostraremos un ejemplo de una serie hipergeométrica no sumable Borel pero sí multisumable. Bajo cierta condición sobre las pendientes del D-módulo hipergeométrico correspondiente, podemos probar que las series hipergeométricas son multisumables. También expondremos algunas ideas para abordar el caso general.

La conjetura de Mond

Guillermo Peñafort Sanchís

Universitat de València

guillermo.penafort@uv.es

Resumen: Al estudiar deformaciones de objetos en determinados contextos, existen dos números, que aquí vamos a llamar μ y τ, que *conjeturalmente*, cumplen la desigualdad

$$\mu \geq \tau.$$

Estos contextos incluyen la teoría de deformaciones de hipersuperficies con singularidad aislada, la teoría de deformaciones de discriminantes de aplicaciones holomorfas finitamente determinadas o, por poner otro ejemplo, el estudio de deformaciones de frontales.
Dado un objeto X, el número μ controla la característica de Euler de un objeto genérico X_ϵ próximo a X, mientras que el número τ es el número de parámetros necesario para construir todas las deformaciones de X. En esta charla intentaremos dar una visión general de la conjetura $\mu \geq \tau$, relacionándola con la propiedad de Cohen Macaulay.

Duality of divisors and curves on Mori dream spaces

Luis José Santana Sánchez, Maria Chiara Brambilla, Olivia Dumitrescu, and Elisa Postinghel

Universidad de Valladolid

luisjose.santana@uva.es

Abstract: Let X be a normal $\mathbb{Q}$-factorial variety with zero irregularity. In [2], Payne addresses a duality problem between the cone of divisors and the cone of curves for toric varieties. Namely, on one hand, for $0 \leq k \leq n-1$, we define D_k to be the cone of numerical classes of effective divisors on X whose stable base locus has codimension larger than k and we set $\mathcal{D}_k$ to be its closure in $\mathrm{N}^1(X)_{\mathbb{R}}$. This gives the filtration

$$\mathrm{Nef}(X) = \mathcal{D}_{n-1} \subseteq \cdots \subseteq \mathcal{D}_1 \subseteq \mathcal{D}_0 = \overline{\mathrm{Eff}(X)}.$$

On the other hand, we denote by $\mathcal{C}_k$ the closure of the cone in $N_1(X)_{\mathbb{R}}$ generated by classes of curves moving in a family that sweeps out an $(n-k)$-dimensional subvariety of X. Thus, we obtain the following filtration for the cone of pseudo-effective curves:

$$\overline{\mathrm{N}E(X)} = \mathcal{C}_{n-1} \supseteq \cdots \supseteq \mathcal{C}_1 \supseteq \mathcal{C}_0.$$

It is known that $\mathcal{C}_{n-1}$ and $\mathcal{C}_0$ are dual to $\mathcal{D}_{n-1}$ and $\mathcal{D}_0$, respectively, under the standard intersection product. In general, duality for intermediate cones is not expected. For toric varieties Payne proved that we reach duality if we also consider curves sweeping out varieties in some small modification of X. We call this *weak duality*. Later, in [1], Choi proved that this weak duality holds for Mori dream spaces and asked if, as the pseudo-effective cone, the $\mathcal{D}_k$ cones are also polyhedral.
In this talk we give a positive answer to Choi's question, by relating the filtration of $\overline{\mathrm{Eff}(X)}$ to the Mori chamber decomposition of X.

Referencias

[1] S. R. Choi (2012). Duality of the cones of divisors and curves. Math. Res. Lett. 19, no. 2, 403-416.

[2] S. Payne (2006). Stable base loci, movable curves, and small modi
cations, for toric varieties. Math. Z. 253, no. 2, pp. 421-431.

Visualizing the valuative Nagata conjecture: A geometric approach

Carlos-Jesús Moreno-Ávila, C. Galindo, F. Monserrat and J.-J. Moyano-Fernández

Universitat Jaume I

carlosjesus.moreno@uva.es

Abstract: The Nagata conjecture arises as a result of the negative answer to the 14th Hilbert problem [5]. Despite many efforts this conjecture remains open after more than 60 years. There exist several reformulations using interesting objects in Algebraic Geometry as Seshadri constants, Mori cone and irrational nef divisors. Recently, a valuative Nagata conjecture has been stated in [2, 4]. This conjecture involves a (real) plane valuation ν of the projective plane and a constant named Seshadri-type constant, denoted $\hat{\mu}(\nu)$ and introduced in [1]. In addition, the valuative Nagata conjecture implies the Nagata conjecture and asymptotic evidences of its trueness in some particular cases have been proved in [4]. In this talk we introduce a natural Seshadri constant for a smooth projective surface S, a nef divisor on S and a divisorial plane valuation of S and we show some of their properties. These results will help us to give several equivalent geometrical statements to the valuative Nagata conjecture for divisorial plane valuations.
This talk is based on a joint work [3] with C. Galindo, F. Monserrat and J.-J. Moyano-Fernández.

Referencias

[1] S. Boucksom, A. Küronya, C. Maclean, T. Szemberg (2015). Vanishing sequences and Okounkov bodies. Math. Ann., 361, 811–834.

[2] M. Dumnicki, B. Harbourne, A. Küronya, J. Roé, T. Szemberg (2017). Very general monomial valuations of $\mathbb{P}^2$ and a Nagata type conjecture. Comm. Anal. Geom., 25, 125–161.

[3] C. Galindo, F. Monserrat, C.-J. Moreno-Ávila, J.-J. Moyano-Fernández (2022). On the valuative Nagata conjecture, *arXiv:2208.11041*.

[4] C. Galindo, F. Monserrat, J. J. Moyano-Fernández (2018). Minimal plane valuations. J. Alg. Geom., 27, 751–783.

[5] M. Nagata (1959). On the 14-th problem of Hilbert. Amer. J. Math., 81, 766–772.

On automorphism groups and twisting theory for smooth plane curves

Eslam Badr

American University in Cairo

eslammath@aucegypt.edu

Abstract: Automorphism groups of smooth plane curves play a crucial role in various areas of mathematics, including algebraic geometry and topology. On the other hand, the study of the set of twists of a curve has been proven to be really useful for a better understanding of the arithmetic behavior of the curve. This talk aims to explore automorphism groups of smooth plane curves and their twists. We will discuss the intricate interplay between the geometry of smooth plane curves and their automorphisms, focusing on the significant role played by twists in uncovering the underlying arithmetics. Moreover, the talk will offer insights into ongoing research in the field, presenting results and open problems that contribute to a deeper understanding of automorphism groups and their twists in smooth plane curves, and more generally for smooth projective hypersurfaces.