Sesión Especial 25

New trends and perspectives in fluid dynamics

Organizadores:

- Diego Alonso-Orán (Universidad de La Laguna)
- Rafael Granero-Belinchón (Universidad de Cantabria)
- Daniel Lear (Universidad de Cantabria)

Descripción:
Esta sesión se centra en el análisis matemático de una amplia clase de ecuaciones en derivadas parciales (EDP) que describen diversos fenómenos físicos dentro del campo de la dinámica de fluidos. Desde un punto de vista matemático, estas EDP son a menudo ecuaciones integro-diferenciales hiperbólicas/parabólicas que suelen combinar efectos no lineales y no locales, lo que supone un reto tanto analítico como computacional. El papel de la comunidad matemática es analizar estas EDP y desarrollar nuevas herramientas matemáticas para entender la interacción de los diferentes efectos, y capturar los fenómenos físicos subyacentes. Algunas de las cuestiones que se abordarán a lo largo de la sesión serán: la estabilidad de las soluciones, la existencia de singularidades en tiempo finito así como el comportamiento asintótico de las mismas.

Programa

Jueves, 25 de enero:

11:30 – 11:55	Antonio J. Fernández (Universidad Autónoma de Madrid) *A Schiffer-type problem with applications to stationary Euler flows*
11:55 – 12:20	Francesco Fanelli (Basque Center for Applied Mathematics) *Rigorous derivation of reduced models for ocean dynamics*
12:20 – 12:45	Francisco Mengual (Max Planck Institute for Mathematics) *Sharp nonuniqueness of admissible solutions for the 2D Euler equation*
12:45 – 13:10	Niklas Knobel (Karlsruhe Institute of Technology) *On the Sobolev Stability Threshold for the 2D MHD Equations with Horizontal Magnetic Dissipation*

16:00 – 16:25	David N. Reynolds (Universidad de Granada) *Coupled Schrödinger-Lohe Systems of Quantum Synchronization*
16:25 – 16:50	Elena Salguero (Max Planck Institute for Mathematics) *Dynamics of turbulent flow from the perspective of Kolmogorov's Theory of Turbulence*
16:50 – 17:15	Oscar Dominguez (Universidad Complutense de Madrid) *Sparse stability for Euler equations*
17:15 – 17:40	Eduardo García-Juárez (Universidad de Sevilla) *The immersed inextensible filament problem*
17:40 – 18:05	Omar Sanchez (ICMAT) *Unbounded solutions for the Muskat problem*

Viernes, 26 de enero:

11:30 – 11:55	Luis Martinez-Zoroa (University of Basel) *Blow-up for the 3D incompressible Euler equations*
11:55 – 12:20	David Poyato (Universidad de Granada) *Heterogeneous Wasserstein gradient flows*
12:20 – 12:45	Jaemin Park (University of Basel) *Stability of stratified density under incompressible flow*
12:45 – 13:10	Bernhard Kepka (Hausdorff Center for Mathematicsl) *Rotating solutions to the incompressible Euler-Poisson equation with external particle*
13:10 – 13:35	Arghir Zarnescu (Basque Center for Applied Mathematics) *On the dynamics of many small bodies in a fluid*

A Schiffer-type problem with applications to stationary Euler flows

Antonio J. Fernández

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Abstract: *If on a smooth bounded domain* $\Omega \subset \mathbb{R}^2$ *there is a nonconstant Neumann eigenfunction* u *that is locally constant on the boundary, must* Ω *be a disk or an annulus?* This question can be understood as a weaker analog of the well-known Schiffer conjecture, in that the function u is here allowed to take a different constant value on each connected component of $\partial\Omega$ yet many of the known rigidity properties of the original problem are essentially preserved. In this talk we provide a negative answer by constructing a family of nontrivial doubly connected domains Ω with the above property. Then, we will show how our construction implies the existence of continuous, compactly supported stationary weak solutions to the 2D incompressible Euler equations which are not locally radial.

The talk is based on a joint work with Alberto Enciso, David Ruiz and Pieralberto Sicbaldi.

Rigorous derivation of reduced models for ocean dynamics

Francesco Fanelli

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Abstract: In this talk, we review some recent results about singular perturbation problems for systems of PDEs describing the dynamics of large-scale incompressible flows. From the physical standpoint, there are two main factors to take into account: the action of gravity and the rotation of the Earth. Thus, the primitive system we consider is given by the density-dependent incompressible Navier-Stokes equations written in a rotating reference frame. The goal is to understand the asymptotic dynamics in the fast rotation (i.e. low Rossby number) limit. We will show convergence to suitable reduced systems in two qualitatively different regimes: the quasi-homogeneous regime and the fully non-homogeneous regime. In passing, we will discuss how the mathematical analysis is able to capture well-known physical phenomena, like the Taylor-Proudman theorem and the Ekman pumping effect.

This talk is based on joint works with Isabelle Gallagher, Dimitri Cobb and Marco Bravin.

Sharp nonuniqueness of admissible solutions for the 2D Euler equation

Francisco Mengual

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Abstract: A weak solution to the Euler equation is called admissible if it does not increase the energy. These weak solutions coincide with a strong solution as long as the latter exists. In this talk we address the question of what is the threshold regularity at $t = 0$ for which uniqueness of admissible solutions fails in 2D. We will show, by means of the convex integration method, sharpness of the weak-strong uniqueness principle, as well as sharpness of the Yudovich proof of uniqueness in the class of bounded admissible solutions.

On the Sobolev Stability Threshold for the 2D MHD Equations with Horizontal Magnetic Dissipation

Niklas Knobel

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Abstract: We consider the evolution of the magnetohydrodynamic (MHD) equations in a periodic channel near a combination of Couette flow and constant magnetic field. We study the partial dissipation regime with full viscous and only horizontal magnetic dissipation. In particular, we show that this regime behaves qualitatively differently than both the fully dissipative and the non-resistive setting. For this data we establish an upper bound on the Sobolev stability threshold.

This is based on joint work with C. Zillinger.

Coupled Schröodinger-Lohe Systems of Quantum Synchronization

David Nicholas Reynolds

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Abstract: In this talk based off a joint work with Paolo Antonelli (GSSI) we will discuss some basics of synchronization dynamics. Then we will introduce a variation on the Schroödinger-Lohe model for quantum synchronization. The model is described by a system of Schroödinger equations, coupled through nonlinear, non-Hamiltonian interactions that drive the system towards phase synchronization. Moreover, interaction strength between different wave functions is regulated through intrinsic parameters θ_j, that follow the Cucker-Smale communication protocol. Unlike the original Schroödinger-Lohe system, where the interaction strength was assumed to be uniform, in the variant we consider the total mass of each quantum oscillator is allowed to vary in time. These extended models yield configurations exhibiting phase, but not space, synchronization. The results are mainly based on the analysis of the ODE systems arising from the correlations, control over the well known Cucker-Smale dynamics, and the dynamics satisfied by the quantum order parameter.

Dynamics of turbulent flow from the perspective of Kolmogorov's Theory of Turbulence

Elena Salguero

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Abstract: In 1941, Kolmogorov developed the celebrated "1941 Theory of Turbulence" in a series of articles where he described the dynamics of turbulent flow from a statistical point of view. Specifically, he characterized turbulent motion in certain regimes through a system of non-linear PDEs, where the unknowns are interpreted as statistical averages. In this talk, we will study the existence of solutions for this system in the framework of fractional Sobolev spaces.

Sparse stability for Euler equations

Oscar Dominguez

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Abstract: A friendly introduction to new tools we use to study energy conservation of solutions to Euler equations of fluid dynamics. In particular, we develop sparse decompositions and extrapolation techniques.

The talk is based on joint work with M. Milman.

The immersed inextensible filament problem

EDUARDO GARCÍA-JUÁREZ

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Abstract: We consider the analysis of a prototypical model for vesicles and red blood cells. More specifically, we study the problem of a one-dimensional inextensible closed filament immersed in a two-dimensional Stokes fluid. The elasticity of the filament is modeled via a bending force corresponding to the Willmore energy. We will derive the boundary integral formulation of the problem and show that the evolution is governed by a parabolic PDE coupled with an elliptic one for the tension, proving its well-posedness in appropriate spaces.

This is a joint work with P.-C. Kuo and Y. Mori.

Unbounded solutions for the Muskat problem

Omar Sanchez

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Abstract: In this talk we show the local existence of solutions of the form $x^2 + ct + g$ for the Muskat problem in the stable regime. We use standard energy methods to obtain a polynomial bound for the energy of the solution in Sobolev spaces $H^s(\mathbb{R})$. It is important to note that this type of solutions are unbounded and have a quadratic growth at infinity.

Blow-up for the 3D incompressible Euler equations

Luis Martinez-Zoroa

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Abstract: The formation of singularities for the incompressible 3D Euler equations is a topic of great importance in fluid dynamics, which has seen many important developments in the last few years. In this talk, we will talk about two recent results regarding blow-up for the incompressible 3D Euler equations: First, we will discuss a non self-similar blow-up in the axi-symmetric without swirl case. Second, we discuss a blow-up for forced incompressible 3D Euler, with a force that is half a derivative above the well-posedness threshold.

Heterogeneous Wasserstein gradient flows

David Poyato

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Abstract: Over the past ten years, optimal transport has become a fundamental tool in Calculus of Variations and Analysis of PDEs. The so called Wasserstein distance provides a new notion of distance between probability distributions and has triggered robust tools to study stability and long time behavior of PDEs with measure-valued solutions, through the theory of Wasserstein gradient flows initiated by L. Ambrosio, N. Gigli and G. Savaré.
In this talk, I will introduce a novel optimal transport distance on the space of probability measures over a fiber bundle. Its associated transport cost is set as to penalize transport from one fiber to another, so that the resulting optimal transport problem becomes constrained to happen along fixed fibers. For simplicity, I will illustrate the construction in the Euclidean case $\mathbb{R}^d \times \mathbb{R}^d$, where the quadratic cost is penalized in the second component. Despite the degeneracy of the cost, we prove that the space $\mathcal{P}_{2,\nu}(\mathbb{R}^{2d})$ of probability measures with fixed marginal $\nu \in \mathcal{P}(\mathbb{R}^d)$ in the second component, endowed with the fibered Wasserstein distance $W_{2,\nu}$ solving the above optimal transport problem, is a Polish space with a nice weak Riemannian structure reminiscent of the one proposed by F. Otto for the classical quadratic Wasserstein space.
First, I will present a general theory of gradient flows with respect to the new metric, which extends previous results by L. Ambrosio, N. Gigli and G. Savaré for the quadratic Wasserstein space. Second, I will showcase several applications that identify a novel fibered gradient flow structure on a large class of evolution PDEs with heterogeneities ranging from heterogenous multi-agent systems and multispecies models arising in collective dynamics. Finally, I shall employ this tool to derive for the first time quantitative convergence rates to equilibrium in the so called Cucker-Smale alignment model with weakly singular interactions.

This is a joint work with J. Pezek (University of Warsaw).

Stability of stratified density under incompressible flow

Jaeming Park

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Abstract: In this talk, I will discuss asymptotic stability in the incompressible porous media equation in a periodic channel. It is well known that a stratified density, which strictly decreases in the vertical direction, is asymptotically stable under sufficiently small, smooth perturbations. We achieve improvements in the regularity assumptions on the perturbation and in the convergence rate. We apply a similar idea to the Stokes transport system. Instead of relying on the linearized equations, we directly address the nonlinear problem, and the decay of solutions will be obtained from the gradient flow structure of the equation.

Rotating solutions to the incompressible Euler-Poisson equation with external particle

Bernhard Kepka

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Abstract: The shape of fluid objects due to the combination of rotational and self-gravitating forces is a classical research field which has been extensively considered for different fluid models, in particular for the incompressible Euler equations.
In this talk we consider an incompressible, two-dimensional fluid body, together with a self-induced attractive force. Furthermore, it is perturbed by an external particle with small mass. The shape of the body is assumed to be close to the unit disk, changing due to the interaction with the particle. In addition, the whole configuration rotates around its center of mass.
We construct solutions to the incompressible Euler-Poisson equation, which are associated with steady states in a rotating coordinate system, via perturbative methods. Moreover, we study a large class of internal motion of the fluid body.
In addition, if time allows we give an account of a work in progress consisting of the corresponding problem in three dimensions.

On the dynamics of many small bodies in a fluid

Arghir Zarnescu

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Abstract: We consider the motion of N rigid bodies – compact sets $(\mathcal{S}_\varepsilon^1, \cdots, \mathcal{S}_\varepsilon^N)_{\varepsilon>0}$ – immersed in a viscous incompressible fluid contained in a domain in the Euclidean space $\mathbb{R}^d$, $d = 2, 3$. We show the fluid flow is not influenced by the presence of the infinitely many bodies in the asymptotic limit $\varepsilon \to 0$ and $N = N(\varepsilon) \to \infty$ as soon as

$$\mathrm{diam}[\mathcal{S}_\varepsilon^i] \to 0 \text{ as } \varepsilon \to 0, \ i = 1, \cdots, N(\varepsilon).$$

The result depends solely on the geometry of the bodies and is independent of their mass densities. Collisions are allowed and the initial data are arbitrary with finite energy.

This is joint work with Eduard Feireisl and Arnab Roy.