Sesión Especial 24

New trends in Linear Algebra and Matrix Analysis

Organizadores:

- Ángeles Carmona (Universitat Politècnica de Catalunya)
- Carlos Marijúan (Universidad de Valladolid)
- Alicia Roca (Universidad de Valencia)

Descripción:
La sesión presentará una amplia gama de trabajos actuales y desarrollos emergentes en el ámbito del álgebra lineal, análisis matricial y sus aplicaciones. Entre los temas que abordaremos en esta sesión, se incluyen, aunque no se limitan a ellos: Formas compañeras, problemas espectrales inversos, matrices estructuradas, resolución de grandes sistemas (métodos iterativos, precondicionamiento, etc), inversas generalizadas, teoría de control, matrices racionales, matrices polinomiales, ecuaciones matriciales, teoría espectral de matrices, pseudoespectros, teoría del potencial discreto o desarrollo de software.

Programa

JUEVES, 25 de enero:

11:30 – 12:00	Carlos Marijúan (Universidad de Valladolid) *Realizability and universal realizability of nonreal spectra of size 6 and trace zero*
12:00 – 12:30	Julio Moro (Universidad Carlos III) *Structured perturbation of eigenvalues of symplectic and Hamiltonian matrices*
12:30 – 13:00	Andrés M. Encinas (Universitat Politècnica de Catalunya) *On the spectrum of bisymmetric Jacobi matrices with periodic coefficients*
13:00 – 13:30	Miriam Pisonero (Universidad de Valladolid) *NIEP, matrices diagonalmente dominantes y digrafos pesados*
16:00 – 16:30	M.J. Jiménez (Universitat Politècnica de Catalunya) *Improving Electrical Impedance Tomography troughh discrete techniques*
16:30 – 17:00	José Mas (Universitat Politècnica de València) *Preconditioning linear systems with V-AISM*
17:00 – 17:30	Ana Marco (Universidad de Alcalá) *Accurate bidiagonal decompositions of some structured totally positive matrices of arbitrary rank*
17:30 – 18:00	Juan Manuel Peña (Universidad de Zaragoza) *On optimal properties related to Total Positivity*

VIERNES, 26 de enero:

11:30 – 12:00	Fernando de Terán (Universidad Carlos III) *On the consistency of the matrix equation $X^\top AX = B$ when B is either symmetric or skew-symmetric*
12:00 – 12:30	Silvia Marcaida (Universidad del País Vasco) *Filters connecting dynamical systems*
12:30 – 13:00	Gorka Armentia (Universidad del País Vasco) *Título*
13:00 – 13:30	Alicia Roca (Universitat Politècnica de València) *Existence of polynomial matrices with some prescribed rows, degree and (part of) eigenstructure*

Realizability and universal realizability of nonreal spectra of size 6 and trace zero

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Abstract: The *Nonnegative Inverse Eigenvalue Problem* (NIEP) consists of the characterization of the lists of complex numbers that are spectra of nonnegative matrices. We say that a list $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ is *realizable* if it is the spectrum of a nonnegative matrix. We say that the realizable list Λ is *universally realizable* if, for every possible Jordan canonical form allowed by Λ, there is a nonnegative matrix with spectrum Λ. The *Universal Realizability Problem* (URP) consists of the characterization of the lists that are universally realizable.
In terms of n, the NIEP is completely solved only for $n \leq 4$, and for $n = 5$ with trace zero. It is clear that for $n \leq 3$ the concepts of universally realizable and realizable are equivalent. The URP is also completely solved for $n \leq 4$ and for $n = 5$ with trace zero in the real case, and partially solved for $n = 5$ with trace zero in the nonreal case. These solutions are different to the NIEP. In this talk we study the realizability and the universal realizability of nonreal spectra of size 6 and trace zero. We use techniques from Graph Theory and Linear Algebra.

References

[1] J. Torre-Mayo, M.R. Abril-Raymundo, E. Alarcia-Estévez, C. Marijuán, M. Pisonero (2007). The nonnegative inverse eigenvalue problem from the coefficients of the characteristic polynomial. EBL digraphs. Linear Algebra Appl. 426, 729–773.

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Acknowledgments: Work partially supported by grant PID2022-138906NB-C21 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe", by the "European Union".

Structured perturbation of eigenvalues of symplectic and Hamiltonian matrices

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Abstract: For certain matrix structures, perturbations which preserve that structure modify eigenvalues very differently from how arbitrary perturbations do. Two such families of matrices are symplectic and Hamiltonian ones. In this talk a detailed first-order structured perturbation analysis is presented for both classes. One of the main features of the analysis is framing symplectic perturbations in a multiplicative, instead of additive, way. Expansions for Hamiltonian perturbations are then derived from symplectic ones via the Cayley transform.

On the spectrum of bisymmetric Jacobi matrices with periodic coefficients

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Abstract: The inverse eigenvalue problem for symmetric matrices consists of determining whether for an ordered list of n real numbers there exists a symmetric and irreducible matrix of order n whose spectrum coincides with the given list. Within this general problem, the case of bisymmetric Jacobi matrices occupies a central place, since for each strictly monotone list, there is a unique bisymmetric Jacobi matrix that realizes it. Regardless of their interest in fields such as mechanics or statistics, families of this type of matrices whose spectrum is known are often used as tests for different recovery algorithms of the coefficients of the matrices from spectral data.

Unfortunately, there are very few families of bisymmetric Jacobi matrices whose spectrum is known. Recently, bisymmetric Jacobi matrices whose eigenvalues form a linear or quadratic progression have been characterized, thus unifying many different works over the last hundred years.

In the case of periodic coefficients, very little is known, reduced to small variations of the constant coefficients case (period 1) and to some very specific situations for period 2. In this communication we are concerned with obtaining expressions for the spectrum of bisymmetric Jacobi matrices with periodic coefficients, for periods lower or equal to 3. In all cases, our technique is based on the analysis of difference equations with periodic coefficients and the boundary problems associated with them.

Acknowledgments: This work has been partly supported by the Spanish Research Council (Comisión Interministerial de Ciencia y Tecnología,) under project PID2021-122501NB-I00 and by the Universitat Politècnica de Catalunya under funds AGRUPS-2022 and AGRUPS-2023. Samir Mondal acknowledges funding received from the Prime Minister's Research Fellowship (PMRF), Ministry of Education, Government of India, for carrying out this work.

NIEP, diagonally dominant matrices, and weigthed digraphs

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Abstract: NIEP stands for the acronym in English for Nonnegative Inverse Eigenvalue Problem. This problem involves characterizing the spectra of nonnegative matrices. In this talk, we will take a brief look at what is known about this problem and its variants, showing its interpretation for weighted digraphs. Note that a nonnegative matrix can be viewed as the adjacency matrix of a weighted digraph. This implies that the NIEP is equivalent to characterizing the spectra of weighted digraphs. Finally, we will describe, in terms of combinatorial structure and sign patterns, when a weakly diagonally dominant matrix is invertible.

References

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Acknowledgments: Work (partially) supported by PID2021-122501NB-I00.

Improving Electrical Impedance Tomography discrete techniques

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Abstract: For health issues, Electrical Impedance Tomography (EIT) represents a non-invasive and radiation-free imaging technique for recovering the conductivity distribution inside the body under observation from skin surface measurements. Besides, EIT is known to be a groundbreaking area of research because its low cost and portable instrumentation. It is well known that this problem is severely ill-posed, especially if complex networks are considered. Therefore, new algorithms to overcome this structural difficulty are necessary. Using discrete techniques, we are verifying that the design of stable algorithms for the recovery of conductances can be achieved through an optimization process. As a consequence, we can improve existing EIT techniques for clinical diagnosis.

Preconditioning linear systems with V-AISM

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Abstract:
To solve a linear system $Ax = b$, where A is a nonsingular, large and sparse matrix using iterative methods, the use of preconditioning techniques is fruitful. In this work we study factorized approximate inverse preconditioners that compute explicitly the preconditioner as an approximation of A^{-1}.
We use the Sherman-Morrison formula to obtain an approximate inverse LU preconditioner. The main difference with respect to the AISM preconditioner is the way of applying recursively the inversion formula to obtain a new decomposition of A^{-1}. Then, we use a compact representation of this decomposition to build our proposed preconditioner V-AISM.
The inverse of A may be computed considering a nonsingular matrix A_0 of the same size and two sets of vectors $\{x_k\}_{k=1}^n$ and $\{y_k\}_{k=1}^n$ such that $A = A_0 + \sum_{k=1}^n x_k y_k^T = A_0 + XY^T$, where $X = [x_1\ x_2\ \cdots\ x_n]$ and $Y = [y_1\ y_2\ \cdots\ y_n]$.
Defining $A_k = A_0 + \sum_{i=1}^k x_i y_i^T$, $k = 1, \ldots, n$ we have $A_k = A_{k-1} + x_k y_k^T$ and $A_n = A$. Suppose that $r_1 = 1 + y_1^T A_0^{-1} x_1 \neq 0$. By the Sherman-Morrison formula the matrix $A_1 = A_0 + x_1 y_1^T$ is nonsingular and

$$A_1^{-1} = A_0^{-1} - \frac{1}{r_1} A_0^{-1} x_1 y_1^T A_0^{-1} = A_0^{-1} \left(I - \frac{1}{r_1} x_1 w_1^T \right) = A_0^{-1} V_1$$

where $w_1^T = y_1^T A_0^{-1}$ and $V_1 = I - \frac{1}{r_1} x_1 w_1^T$. If $r_k = 1 + y_k^T A_{k-1}^{-1} x_k \neq 0$, then

$$A_k^{-1} = A_{k-1}^{-1} - \frac{1}{r_k} A_{k-1}^{-1} x_k y_k^T A_{k-1}^{-1} = A_{k-1}^{-1} \left(I - \frac{1}{r_k} x_k w_k^T \right) = A_{k-1}^{-1} V_k,$$

where $w_k^T = y_k^T A_{k-1}^{-1}$ and $V_k = I - \frac{1}{r_k} x_k w_k^T$. Then, $A^{-1} = A_n^{-1} = A_0^{-1} V_1 \cdots V_n$.
To compute the preconditioner, small entries are zeroed in the process. We prove that this process is breakdown-free for M- and H-matrices. Moreover, numerical experiments show that this new preconditioner is efficient and faster than AISM.

Acknowledgments: Work supported by Conselleria de Innovación, Universidades, Ciencia y Sociedad Digital, Generalitat Valenciana (CIAICO/2021/162)

Accurate bidiagonal decompositions of some structured totally positive matrices of arbitrary rank

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Abstract: The fast and accurate computation of bidiagonal decompositions of nonsingular totally positive matrices is an active topic of research in the area on numerical linear algebra. The interest in this subject is partly due to the work of P. Koev, who proved in [1] that, having the bidiagonal decomposition of a nonsingular totally positive matrix computed to high relative accuracy, virtually all linear algebra problems can be solved accurately for that matrix. The extension of these results to the case of singular totally positive matrices was also recently carried out in [2], which motivated the search of fast and accurate algorithms to compute the bidiagonal decompositions of structured totally positive matrices of any rank. In particular, these decompositions were obtained for the case of totally positive Vandermonde-type matrices in [3]. In this talk, we present fast and accurate algorithms to compute the bidiagonal decomposition of some structured totally positive matrices which are also valid when these matrices are singular. The application of our algorithms to the accurate and efficient computation of the eigenvalues of these structured matrices is also considered. Numerical experiments showing the good performance of our approach will be presented.

References

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On optimal properties related to Total Positivity

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Abstract: A matrix whose minors are all nonnegative is called totally positive. A basis of a space U of univariate functions is called totally positive if all its collocation matrices are totally positive. If U has a totally positive basis, then there exists a totally positive basis of U (called B-basis) such that it generates all totally positive bases of U by means of totally positive matrices. A first example of B-basis is the Bernstein basis of polynomials on a compact interval. Several optimal properties have been obtained for B-bases and their collocation matrices as well as for other related matrices (see [1] and [2]). This talk will also present some new contributions to this field, in particular some results related to Wronskian matrices of B-bases.

References

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Acknowledgments: Work (partially) supported by the Spanish research grants PID2022-138569NB-I00 and RED2022-134176-T (MCIU/AEI) and by Gobierno de Aragón (E41_23R).

On the consistency of the matrix equation $X^\top AX = B$ when B is either symmetric or skew-symmetric

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Abstract:
We will review some recent work about the consistency of the matrix equation

$$X^\top AX = B, \tag{1}$$

when B is either symmetric or skew-symmetric. In Equation (1), $A \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{m\times m}$, $X \in \mathbb{C}^{m\times n}$ is the unknown and $X^\top$ is the transpose of X. More precisely, we will show that, for most matrices A:

- If B is symmetric, then (1) is consistent if and only if $\operatorname{rank} B \leq \min\{n - n_A - \frac{\operatorname{rank}(A-A^\mathsf{T})}{2}, \operatorname{rank}(A + A^\mathsf{T})\}$.
- If B is skew-symmetric, then (1) is consistent if and only if $\operatorname{rank} B \leq \min\{n - n_A - \frac{\operatorname{rank}(A+A^\mathsf{T})}{2}, \operatorname{rank}(A - A^\mathsf{T})\}$, where $n_A = \dim(\operatorname{Nul} A \cap \operatorname{Nul} A^\mathsf{T})$.

The main tool that we have used to get such results if the so-called *Canonical form for congruence* of the matrices A and B, that will be reviewed in the talk.

This is based on joint work with Alberto Borobia and Roberto Canogar, from UNED.

Acknowledgments: This work has been partially supported by the *Ministerio de Economía y Competitividad* through grant MTM2015-65798-P, by the *Ministerio de Ciencia, Innovación y Universidades* through grant MTM2017–90682–REDT, by the *Agencia Estatal de Investigación* through grants PID2019-106362GB-I00/AEI/10.13039/501100011033 and RED2022-134176-T, and by the *Comunidad de Madrid* under the Multiannual Agreement with UC3M in the line of Excellence of University Professors (EPUC3M23), and in the context of the V PRICIT (Regional Programme of Research and Technological Innovation).

Filters connecting dynamical systems

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Abstract: Filters connecting two dynamical systems, identified with polynomial matrices $D_1(s), D_2(s)$, are polynomial matrices $F_1(s), F_2(s)$ that satisfy the equation $F_2(s)D_1(s) = D_2(s)F_1(s)$. Filters that connect quadratic systems with the same finite eigenvalues and their partial multiplicities were called coprime filters in [3]. The notion of coprime filters has been extended for systems and their corresponding matrix polynomials of possibly different sizes, ranks or degrees that share the same spectral structure, i.e., the same finite and infinite elementary divisors. Such filters were named spectral filters in [1] and they completely characterize when two polynomial matrices are spectrally equivalent, that is, when they have the same spectral structure. A parametrization of the set of spectral filters connecting two nonsingular polynomial matrices has been defined in [2], where the parameter space is the subset of invertible matrices of the centralizer of any linearization of the reversals with respect to a scalar that is not an eigenvalue of the given matrices. One application of filters is to decouple systems. We have been able to identify the parameter that defines the filters that decouple quadratic systems.

References

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Acknowledgments: Work supported by grant PID2021-124827NB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe" by the "European Union", and by grant GIU21/020 funded by UPV/EHU.

Approximation of pseudospectra of block triangular matrices

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Abstract: Let $A \in \mathbb{C}^{n\times n}$ be unitarily similar to

$$\begin{bmatrix} L & C \\ O & M \end{bmatrix},$$

where $L \in \mathbb{C}^{\ell\times\ell}$, $M \in \mathbb{C}^{m\times m}$ and $C \in \mathbb{C}^{\ell\times m}$. We denote by $\Lambda_\varepsilon(A)$ the ordinary ε-pseudospectrum of A, which can be characterized as

$$\{z \in \mathbb{C} \colon \|(zI_n - A)^{-1}\|^{-1} \leqslant \varepsilon\},$$

where $\|\cdot\|$ stands for the spectral norm or 2-norm. The goal of this talk is to give some outer and inner bounds for $\Lambda_\varepsilon(A)$ in terms of the pseudospectra of L and M. More precisely, the results for the outer bounds of the ε-pseudospectrum of A involve expressions of the form

$$\Lambda_\varepsilon(A) \subseteq \Lambda_{g(\varepsilon)\varepsilon}(L) \cup \Lambda_{g(\varepsilon)\varepsilon}(M),$$

for some function g. When it comes to the inner bounds of the ε-pseudospectrum of A, these are of the form

$$\Lambda_{f(\varepsilon)\varepsilon}(L) \subseteq \Lambda_\varepsilon(A) \qquad \text{and} \qquad \Lambda_{\widehat{f}(\varepsilon)\varepsilon}(M) \subseteq \Lambda_\varepsilon(A),$$

for some functions f and $\widehat{f}$.

Existence of polynomial matrices with some prescribed rows, degree and (part of) eigenstructure

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Abstract: A solution to the row-completion problem of a polynomial matrix, when the degree and eigenstructure are prescribed is given in [1]. With the help of the First Frobenius companion form of a polynomial matrix, we turn the problem into a pencil completion problem, and solve it using the solution to the pencil row-completion problem obtained in [2].
We have also solved the problem when the degree and only part of the eigenstructure are prescribed, leading to different problems depending on which ones of the invariants that form the eigenstructure are prescribed. With appropriate combinatorial effort, we have completed the study for every possible particular case of prescription of invariants. We will show the solution to some of the cases.

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Acknowledgments: Work partially supported by grant PID2021-124827NB-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe", by the "European Union".