Sesión Especial 23

Mathematical Optimization: Methods and Applications

Organizadores:

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Summary:
Mathematical Optimization deals with the minimization or maximization of one or more functions on a given feasible region. The increasing need of making decisions with large and complex datasets makes it necessary to develop new models and techniques able to deal with that information. In this session we will provide an overview of some of the recent advances in the analysis and design of mathematical models and algorithms to solve challenging problems in different fields, from logistics and transportation to machine learning.

Programa

Jueves, 25 de enero:

16:00 – 16:30 Ricardo Gázquez (Universidad Carlos III de Madrid)
The Cooperative Maximum Capture Facility Location Problem

16:30 – 17:00 Sergio Anglada (Universidad de Zaragoza)
Effect of capacity constraints and packing on the vertex separator problem

17:00 – 17:30 Alberto Torrejón (Universidad de Sevilla)
Conexiones con precios óptimos

17:30 – 18:00 Miguel A. Pozo (Universidad de Sevilla)
The biobjective minimum-cost perfect matching problem and Chinese postman problem

Viernes, 26 de enero:

11:30 – 12:00 Justo Puerto (Universidad de Sevilla)
Conic Linear Programming Games

12:00 – 12:30 Aitor Hernández (Universidad de Zaragoza)
A bilevel optimisation model for the Rank Pricing Problem with Tie

12:30 – 13:00 Francisco Temprano (Universidad de Sevilla)
A fresh view on the Minimum Normalized Cuts Problem

13:00 – 13:30 Víctor Blanco (Universidad de Granada)
The Minimum Cardinality $\boldsymbol{\alpha}$-Mediated Graph Problem

The Cooperative Maximum Capture Facility Location Problem

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Abstract: In the Maximum Capture Facility Location (MCFL) problem with a binary choice rule, a company intends to locate a series of facilities to maximize the captured demand, and customers patronize the facility that maximizes their utility. In this talk, we generalize the MCFL problem assuming that the facilities of the decision maker act cooperatively to increase the customers' utility over the company. We propose a utility maximization rule between the captured utility of the decision maker and the opt-out utility of a competitor already installed in the market. Furthermore, we model the captured utility by means of an Ordered Median function (OMf) of the partial utilities of newly open facilities. We name this problem "the Cooperative Maximum Capture Facility Location problem" (CMCFL). The OMf serves as a means to compute the utility of each customer towards the company as an aggregation of ordered partial utilities, and constitutes a unifying framework for CMCFL models.
We introduce a multiperiod non-linear bilevel formulation for the CMCFL with an embedded assignment problem characterizing the captured utilities. For this model, two exact resolution approaches are presented: a MILP reformulation with valid inequalities and an effective approach based on Benders' decomposition. Extensive computational experiments are provided to test our results with randomly generated data and an application to the location of charging stations for electric vehicles in the city of Trois-Rivières, Quèbec, is addressed.

Effect of capacity constraints and packing on the vertex separator problem

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Abstract: Graph disconnection problems are well known optimization problems in the literature and there exist multiple variants of them. Given an undirected connected graph, the Vertex Separator Problem (VSP) looks for the smallest subset of vertices, called *separator*, whose removal disconnects the graph into two connected components of roughly equal size. In this work we focus on the study of two variants of the VSP with capacity constraints.
The first variant consists of finding the smallest separator such that the number of *connected components* in the subgraph generated after the vertex removal is limited, and the cardinality of each connected component cannot be greater than a fixed bound. This variant motivates a second VSP variant where we consider the packing of the connected components into a limited number of bins (called *shores*) with a known capacity. This second VRP variant has been introduced by Furini et al. [1].
We introduce a new single-level mathematical formulation for each of these VSP variants, and discuss a computational experience using benchmark graphs. The main purpose is to analyze how the two capacity constraints impact on solving the two VSP variants.

References

[1] F. Furini, I. Ljubic, E. Malaguti, P. Paronuzzi (2021). Casting Light on the Hidden Bilevel Combinatorial Structure of the Capacitated Vertex Separator Problem. Opperations Research, 70(4), 2399-2420.

Conexiones con precios óptimos

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Resumen: En una red dada, las conexiones entre puntos pueden poseerse y, por tanto, venderse. Esta situación abre un amplio abanico de problemas combinatorios. Sea $G = (V, E)$ un grafo dado cuyo conjunto de aristas está particionado en un conjunto R de aristas rojas y un conjunto B de aristas azules, y supongamos que las aristas rojas están ponderadas y contienen un árbol de spanning de G. El juego del árbol de spanning mínimo de Stackelberg (`StackMST`) es el de poner precio a las aristas azules de tal forma que se maximice el peso total de las aristas azules seleccionadas en un árbol de spanning mínimo del grafo resultante. En esta charla presentamos diferentes mejoras del estado del arte para el StackMST basadas en las propiedades del Problema del Árbol Mínimo de Expansión y la optimización binivel. Establecemos una comparación teórica y empírica entre estas nuevas formulaciones.

Agradecimientos: Esta investigación ha sido apoyada por el proyecto "*Retos de la optimización combinatoria en los nuevos modelos de redes complejas y ciencia de datos*" (Proyecto I+D+i Junta de Andalucía, referencia P18-FR-1422) "*Nuevos resultados sobre problemas de diseño y optimización en redes complejas: Aplicaciones al diseño de ciudades inteligentes*" (Proyecto I+D+i FEDER Andalucía, referencia US-1256951) y "*Optimización en problemas de ciencia de datos y diseño de redes: Large scale network models meet optimization and data science tools*" (Plan Estatal 2017-2020 Generación Conocimiento - Proyectos I+D+i, referencia PID2020-114594GB-C21). Se agradece este apoyo.

The biobjective minimum-cost perfect matching problem and Chinese postman problem.

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Abstract: In this paper, we address the biobjective versions of the perfect matching problem (PMP) and the Chinese postman problem (CPP). Both problems are solved by means of integer formulations or separating blossom inequalities, exploiting the PMP relationship with the CPP. In both cases, we first find the set of supported nondominated solutions and then we use them to obtain the nonsupported ones. The set of supported nondominated solutions are obtained solving scalarized integer formulations. To obtain the sets of nonsupported solutions, we resort to solving lexicographic problems based on adding additional linear constraints to the original problems. For this reason, we also characterize the combinatorial structure of the PMP vertices with one or two additional constraints. We also investigate when it is possible to use the PMP to solve CPP in the biobjective case. We report computational experiments comparing the different approaches and formulations based on different types of graphs with up to 700 nodes.

References

[1] Pozo, M.A., Puerto, J. and Roldán, I. (2023), The biobjective minimum-cost perfect matching problem and Chinese postman problem. Intl. Trans. in Op. Res.. https://doi.org/10.1111/itor.13363

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Conic linear programming games

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Abstract: In this paper we consider allocation problems derived from conic optimization problems that arise analyzing cooperation in several situations. Specifically, we prove that cooperation can be enforced in situations where individual decision-makers define its worth as the optimal solution of a conic linear program. We construct a generic cooperative game with characteristic function induced by the optimal value of those programs and prove that the game is superadditive and balanced. Then, we apply this framework to two challenging situations: Markowitz model of portfolio selection and generic mixed binary non-convex quadratic minimization problems. In the former case, we give sufficient conditions ensuring cooperation among investors and in the latter, we prove, under mild conditions, balancedness of the corresponding cooperative game providing explicitly core allocations. This last model includes as particular instances several well-known combinatorial optimization games as location, covering and traveling salesman games.

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A bilevel optimisation model for the Rank Pricing Problem with Ties

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Abstract: The Rank Pricing Problem with Ties (RPPT) arises from the need to determine the prices of a set of products offered by a company to a set of customers in order to maximize its revenue. Every customer has a budget and is interested in a subset of products which are ranked by preference. In the RPPT customers are allowed to express indifference in terms of preference among products, leading to ties in customer preference lists. Once the prices have been set, each customer will purchase the cheapest product among those they can afford (if any) and satisfy them the most. The RPPT can be modelled from a bilevel optimisation perspective. Bilevel optimisation models involve two decision makers within a hierarchical framework. In the case of the RPPT, at the upper level of the hierarchy, the company establishes the prices of the products and, at the lower level, the customers decide on the product they purchase. To ensure that a bilevel optimisation model is well posed, uniqueness of the optimal solution of the lower level problem is needed. If this is not the case, it is necessary to establish criteria in order to choose one of the optimal solutions. Several approaches may be considered. The optimistic aproach consists of choosing the optimal solution that benefits the most the upper level. On the contrary, the pessimistic approach consists of choosing the optimal solution that benefits the least the upper level. The latter is the appropiate approach when the RPPT is modelled from a bilevel optimisation perspective. In order to address the pessimistic approach, a biobjective function is included in the lower level problem. The multiobjective treatment of this problem results in an exact procedure for solving the RPPT, different from the ones that have been proposed in the literature so far. Extensive computational experiments have been performed to illustrate the competetiveness of this approach.

References

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A fresh view on the Minimum Normalized Cuts Problem

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Abstract: This paper deals with the k-way normalized cut problem in networks. The normalized cut function was defined to solve some issues concerning the interpretability of the minimum cut problem, which is a classical problem in graph theory whose aim is to provide the bipartition that minimizes the number of edges between nodes from different subsets, applied to partitioning and districting problems. Instead of considering just the number of external edges of each subset, the minimum k-way normalized cut problem tries to minimize the external edge density of each subset of a k-partition, also considering the number of internal edges. In addition, the problem can be extended to a weighted graph in order to minimize the sum of external weight density of the subsets. The minimum k-way normalized cut allows us to locate groups of nodes that accumulate a high internal weight density. Considering that these weights can represent a large number of different interesting parameters, this organization is really useful as facility location planning in order to locate in each of the clusters a facility that accumulates a high weight density. We show several applications of the minimum k-way normalized problem to facility location. We present a methodology using mathematical optimization to provide mixed integer linear programming formulations for the problem. The paper also develops a branch-and-price algorithm for the above mentioned problem which scales better than the compact formulations. Extensive computational experiments assess the usefulness of these methods to solve the k-way normalized cut problem over different location problems on large graphs and random graphs. In addition, all methods have been analysed and studied in order to try to improve them as much as possible.

References

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The Minimum Cardinality $\boldsymbol{\alpha}$-Mediated Graph Problem

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Abstract: The use of conic structures in mathematical optimization problems has been widely recognized within the last few years. The development of interior point methods with specialized barriers has allowed to efficiently solve problems involving cones with a high practical interest. This is the case of facility location, power flow, radiotherapy, or supervised classification, among many others. In this work, we analyze suitable representations of the $(p, \boldsymbol{\alpha})$-cone by simpler cones. Apart from providing constructive approaches to simply represent this type of cones, we derive a novel approach to construct minimal representations of the cone as simpler second order cones (which can be efficiently handled by most off-the-shelf optimization solvers). The main structures that we use for this construction are the so-called *mediated graphs* (see e.g., [1, 2]). We prove the one-to-one correspondence between extended representation of a cone and certain mediated graphs, and derive a mathematical optimization formulation to construct minimal representations, the so-called Minimum Cardinality $\boldsymbol{\alpha}$-Mediated Graph Problem. We report the results of some experiments that we run to test our approach. both in terms of its computational complexity and in a new problem that arises in continuous facility location with attraction.

References

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