Sesión Especial 21

Advanced numerical techniques for the solution of differential problems

Organizers:

- Andrés Arrarás (Public University of Navarre)
- Laura Portero (Public University of Navarre)
- Carmen Rodrigo (University of Zaragoza)

Summary:
Both ordinary and partial differential equations appear in several fields of science and technology when mathematical modelling is applied to describe multitude of phenomena. It is well known that the analytical solution of these models is not possible in most of the cases, and, therefore, numerical methods play a crucial role in order to obtain approximations of their solution in a precise and efficient way. The main aim of this session is to deal with the most recent advances in this field within the research community of our country.

Program

THURSDAY, 25th of January:

11:30 – 12:00	Florin Radu (Bergen University) *On splitting schemes for coupled PDEs*
12:00 – 12:30	Íñigo Jimenez-Ciga (Public University of Navarre) *First-order space-time parallel solvers for mathematical models in biology*
12:30 – 13:00	Severiano González-Pinto (University La Laguna) *Mitigating the reduction on the convergence order in the time integration of parabolic problems with splitting methods*
13:00 – 13:30	Etelvina Javierre (University of Zaragoza) *A multiphase porous model of the mechanochemical coupling in tumor growth*

16:00 – 16:30	María González Taboada (University of A Coruña) *A new adaptive mixed FEM for stationary convection-diffusion-reaction problems*
16:30 – 17:00	Virginia Selgas (University of Oviedo) *A Trefftz Discontinuous Galerkin method for the time-harmonic wave propagation in a waveguide*
17:00 – 17:30	Álvaro Pé de la Riva (University of Zaragoza) *Multigrid solvers for isogeometric discretizations of Biot's equations*
17:30 – 18:00	Javier Zaratiegui (University of Zaragoza) *An efficient solver based on multigrid methods for logically rectangular meshes*

FRIDAY, 26th of January:

11:30 – 12:00	Macarena Gómez Mármol (University of Seville) *Numerical Solution of Differential Equations Systems Slow-Fast*
12:00 – 12:30	Laura Saavedra (Polytechnic University of Madrid) *Invariant domain preserving ALE approximation of Euler equations*
12:30 – 13:00	María López-Fernández (University of Malaga) *Convolution Quadrature for the quasilinear subdiffusion equation*
13:00 – 13:30	Dionisio F. Yáñez (University of Valencia) *Subdivision schemes based on local polynomial regression*

On splitting schemes for coupled PDEs

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Abstract: Solving fully coupled partial differential equations (PDEs) is a very common and challenging task nowadays. In this work we will focus on efficient and robust splitting schemes for coupled PDEs. We will begin with a classical splitting scheme, the fixed-stress scheme [4] (the name is comming from applications in poromechanics). We will discuss its optimization, stabilization (convergence) and acceleration [2, 3, 6]. The acceleration will be based on Anderson accelleration [1]. Further, a new family of splitting schemes based on approximate Schur complement will be presented [5]. Finally, non-linear extensions of coupled PDEs will be briefly mentioned.

References

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First-order space-time parallel solvers for mathematical models in biology

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Abstract: In the framework of large and costly integration of differential problems, parallel computing has become a useful strategy, which permits to obtain faster results and avoids the limitations of speed of sequential computation. Concerning time-dependent partial differential equations, space parallelization could be considered as a first approach. However, if a large number of processors is available, space-time parallel methods are significantly more efficient as regards the optimal implementation of parallelization techniques.

In this work, we present four space-time parallel schemes based on the parareal method (cf. [3]). This algorithm considers two propagators, one of them inaccurate and inexpensive that works sequentially, and the other one accurate and expensive, which is implemented in parallel. In particular, we consider splitting techniques for both propagators, which, with a suitable partition of the problem, can be parallelized in space. In this framework, we select two first-order splitting time integrators for the solution of the partitioned problem, namely: the fractional implicit Euler scheme, formulated in [2], and the Douglas-Rachford method, proposed in [1].

We show stability and convergence properties of the resulting methods, together with some applications to reaction-diffusion problems in the context of mathematical biology models which require accurate solution techniques.

References

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Mitigating the reduction on the convergence order in the time integration of parabolic problems with splitting methods.

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Abstract: We consider a technique to mitigate the order reduction in the convergence order (PDE-convergence order) of some splitting methods applied to the time integration of multidimensional parabolic problems under time dependent Robin boundary conditions. The splitting considered is directional and the methods of ADI type (alternating directions implicit), in particular we focus on AMF-W-methods. Interesting references on the subject are [1, Chapter IV], [2, 3].

References

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A multiphase porous model of the mechanochemical coupling in tumor growth

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Abstract: The progression of cancer is characterized by alterations in cellular responses to both chemical and mechanical signals, together with changes in the mechanical properties of the extracellular matrix (ECM) components of the host tissue and the tumor mass [1]. The forces and stresses created during tumor growth compress the blood and lymphatic vessels leading to poor tissue perfusion and resulting in hypoxia and elevated interstitial fluid pressure. This in turn may promote the recruitment of pro-tumor cells, but also impairs the efficacy of drug therapy. In this context, the development of predictive mathematical models are of great importance to help in the understanding of the intricate mechanochemical coupling regulating tumor growth.

In this work, we regard the tissue as a two-dimensional multiphase porous material that consists of the extracellular matrix, tumoral cells, healthy cells and interstitial fluid. The solid matrix is modelled as an elastic material, and the cellular activity is regulated by the solid stress generated by tumor growth and the oxygen availability. We will discuss stable finite element discretizations for the resulting coupled problem, and we will present numerical results that illustrate the growth of tumors under free and constrained conditions.

References

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A new adaptive mixed FEM for stationary convection-diffusion-reaction problems

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Abstract: We study the approximation of a stationary convection-diffusion-reaction model by a non-stationary problem. We propose a numerical method that combines the method of characteristics with an augmented mixed finite element procedure. We show that this scheme has a unique solution, derive a residual-based a posteriori error indicator, and prove its reliability and local efficiency. Finally, we provide some numerical experiments that illustrate the performance of the adaptive algorithm.

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A Trefftz Discontinuous Galerkin method for the time-harmonic wave propagation in a waveguide

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Abstract: We investigate numerically the propagation of time-harmonic waves along an unbounded waveguide for medium and large piecewise constant wavenumbers. We do so based on a Trefftz Discontinuous Galerkin (TDG) formulation which we discretize with the superposition of travelling plane waves.

We first rewrite the problem on a bounded computational domain by using the Neumann-to-Dirichlet map on the artificial walls. This truncated problem is formulated variationally in a DG way, that is, the interelement continuity is imposed weakly within the variational formulation by introducing suitable numerical fluxes. We choose standard numerical fluxes for internal faces and some more exotic numerical fluxes for faces on the truncation boundary. We then get a consistent and coercive formulation which achieves quasi-optimal convergence when discretized with Trefftz elements. We also provide a priori error bounds for the discretization based on plane waves.

The behavior of the numerical solutions and their order of convergence is verified and illustrated in two dimensions with numerical experiments, including for the special case of the Ultra Weak Variational Formulation (UWVF). In particular, these experiments allow us to investigate the instability and ill-conditioning inherent in plane wave-based Trefftz methods, and if this issue can be overcome in practice with suitable regularization techniques.

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Multigrid solvers for isogeometric discretizations of Biot's equations

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Abstract: Isogeometric analysis (IGA) is a numerical technique based on applying spline-type basis functions for both discretization of partial differential equations (PDEs) and the construction of computational domains. Thus, we propose the use of IGA for discretization of the poroelastic Biot's equations that model the soil consolidation process. Given that large-sparse linear systems arise from the discretization of Biot's equations, it is a key point to apply efficient solvers with a desirable robust convergent behavior such as multigrid methods. Indeed, these methods are among the fastest algorithms for the numerical solution of PDEs. At this point, Biot's equations can be solved via coupled or decoupled solvers. In this work, we are interested in the performance of multigrid methods when they are applied as coupled solvers. Hence, we propose the use of monolithic multigrid methods based on coupled and decoupled smoothers. For the latter approach, we will apply an inexact version of the fixed-stress split method as decoupled smoother based on additive Schwarz methods.

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An efficient solver based on multigrid methods for logically rectangular meshes

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Abstract: Multigrid methods are efficient and powerful techniques to solve the large sparse linear systems of equations arising from the discretization of partial differential equations. The reason relies on the fact that they take advantage of two important properties. On one hand, when properly applied to discrete elliptic problems, classical iterative methods have a smoothing effect on the error of the approximate solution after a few iterations. On the other hand, such a smooth error can be well approximated on a coarse grid. Thus, it becomes computationally less expensive to solve the error equation on the coarse grid and then interpolate the solution to the fine mesh in order to correct the original approximation (cf. [3]). In this context, the application of monolithic multigrid methods for problems where logically rectangular meshes are considered plays an important role. This is due to the fact that this type of meshes can take advantage of recent computer architectures that achieve their best performance when structured data are used. In this work, we propose a multigrid solver based on logically rectangular meshes for different problems: multipoint flux approximations of the Darcy problem for single-phase flow in single and double porosity rigid porous media (cf. [1]), and a multipoint stress approximation of the elasticity problem (cf. [2]). Finally, the robustness of the proposed solver is illustrated through a collection of numerical experiments for the aforementioned problems.

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Numerical Solution of Differential Equations Systems Slow-Fast

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Abstract: In different fields of science such as biology, engineering, medicine, etc., phenomena appear that involve several variables whose dynamics occur at different time scales. These phenomena can be modelled using systems of differential equations, where the characteristic times of the variables are different. These systems are known as slow-fast systems.

Most of these systems are quite complex and can contain a large number of equations, making analytical solutions unfeasible. From the point of view of mathematical analysis, a whole theory relating to this type of systems has been studied for some decades, specifically studying systems of a dynamic type and carrying out a study of possible bifurcations, invariants, etc.

We are interested in calculating numerical approximations of the solutions of such problems. From the point of view of numerical analysis, these problems present a number of important difficulties. The fact that they have very different time scales leads us to solve stiff problems, which requires the use of specific methods for this type of problem. Usually, these methods are coupled with conditions on the time step that can be used to maintain stability, i.e. the time step must be chosen to make the scheme stable and not for reasons of accuracy. Therefore, it is necessary to use specific methods designed for this type of problem.

On the other hand, the large number of equations that the system has, leads us to use techniques that reduce the effective calculation in the simulation in order to obtain approximations in reasonable times, in our case we will use reduced order method techniques.

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Invariant domain preserving ALE approximation of Euler equations

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Abstract: Arbitrary Lagrangian Eulerian (ALE) techniques mix the advantages of classical Lagrangian hydrodynamics methods while minimizing their shortcomings. We have developed ALE methods (see [1, 2, 3]) to solve nonlinear hyperbolic systems while preserving invariant domains. Our ALE methods are based on continuous finite elements and explicit time stepping and are stabilized by means of graph-based artificial viscosity. First, we propose a first-order artificial viscosity that does not require any ad hoc parameters and results in precise invariant domain properties and entropy inequalities. Second, we describe a high-order method that preserves the invariant domains by combining the first-order method with an entropy-consistent high-order method via a convex limiting process.
Building upon the previously established discretization framework, I will introduce an explicit Lagrangian approximation technique for the compressible Euler equations. I will illustrate numerically the robustness of those methods on various benchmark problems.

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Convolution Quadrature for the quasilinear subdiffusion equation

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Abstract: We construct a Convolution Quadrature (CQ) scheme for the quasilinear subdiffusion equation and supply it with the fast and oblivious implementation. We find a condition for the CQ to be admissible and discretize the spatial part of the equation with the Finite Element Method. We prove the unconditional stability and convergence of the scheme and find a bound on the error. As a passing result, we also obtain a discrete Grönwall inequality for the CQ, which is a crucial ingredient of our convergence proof based on the energy method. We show numerical results which verify our convergence theory and show computational time reduction when using fast and oblivious quadrature.

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Subdivision schemes based on local polynomial regression

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Abstract: The generation of curves and surfaces from given data is a well-known problem in Computer-Aided Design that can be solved by means of subdivision schemes. They are a powerful tool that allows obtaining new data from the initial one using simple calculations. In some real applications, the initial data are given with noise and interpolatory schemes are not adequate to process them. In this talk, we present some new families of binary univariate linear subdivision schemes using weighted local polynomial regression. We study their properties, such as convergence, monotonicity and polynomial reproduction and show some examples.

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