Sesión Especial 16

Geometría Algebraica entre Aplicaciones y Computación

Organizadores:

- Marina Garrote-López (Max Planck Institute for Mathematics in the Sciences)
- Beatriz Pascual Escudero (Universidad Politécnica de Madrid)
- Josué Tonelli-Cueto (Johns Hopkins University)

Descripción:
La geometría algebraica estudia objetos descritos por polinomios. A medida que estos objetos geométricos incrementan su presencia en aplicaciones (ciencias de la vida, ingeniería, etc.) y las posibilidades de tratarlos computacionalmente se incrementan, la necesidad por comprender, desarrollar e implementar métodos computacionales en geometría algebraica se vuelve más imperiosa. En esta sesión, se presentarán algunos de los últimos desarrollos en geometría algebraica aplicada y computacional.

Programa

Jueves, 25 de enero:

11:30 – 12:00	Marta Casanellas Rius (Universitat Politècnica de Catalunya) *Simple equations for phylogenetic algebraic varieties*
12:00 – 12:30	Roser Homs Pons (Centre de Recerca Matemàtica) *A novel algebraic approach to time-reversible evolutionary models*
12:30 – 13:00	Liam Solus (KTH Royal Institute of Technology) *The Geometry of Colored Gaussian DAG Models*
13:00 – 13:30	Angélica Torres (Max Planck Institute for Mathematics in the Sciences) *The Voronoi diagram of codimension one varieties under polyhedral norms*
16:00 – 16:30	Cordian Riener (University of Tromsø) *The Wonderful Geometry of the Vandermonde Map*
16:30 – 17:00	Máté L. Telek (University of Copenhagen) *On properties of the signed reduced A-discriminant*
17:00 – 17:30	Philippe Gimenez (IMUVa, Universidad de Valladolid) *Gluing and splitting of homogeneous toric ideals*
17:30 – 18:00	Miruna-Stefana Sorea (Lucian Blaga University of Sibiu, Romania) *Combinatorial study of morsifications of real univariate singularities*

VIERNES, 26 de enero:

11:30 – 12:00	Laureano Gonzalez-Vega (CUNEF Universidad) *On the eigenvalues of Q-matrices and P-matrices*
12:00 – 12:30	Alexandru Iosif (Universidad Rey Juan Carlos de Madrid) *Duality in mass-action networks: A step closer to a new case of the Global Attractor Conjecture?*
12:30 – 13:00	Pedro R. López-Gómez (Universidad de Cantabria) *Measure-preserving mappings from the unit cube to spheres and projective spaces*
13:00 – 13:30	Eduardo Sáenz-de-Cabezón (Universidad de La Rioja) *Importance measures of multi-state systems via monomial ideals*

Simple equations for phylogenetic algebraic varieties

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Abstract: Phylogenetics studies the evolutionary relationships among species using their molecular sequences, which are represented on a phylogenetic tree or network. A Markov process on a phylogenetic tree or network parametrizes an algebraic variety, the so-called phylogenetic variety. One can use a general Markov model or one of its submodels: G-equivariant models with symmetries defined by a group G.
Algebraic geometry has been used for phylogenetic reconstruction. Since G-equivariant models have fewer parameters than a general Markov model, their phylogenetic varieties are defined by more equations and these are usually hard to find. We will see that we can easily derive equations for G-equivariant models from the equations of a phylogenetic variety evolving under a general Markov model.

A novel algebraic approach to time-reversible evolutionary models

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Abstract: Algebraic tools have been proven to be useful in phylogenetic reconstruction and model selection by means of the study of phylogenetic invariants. However, up to now, the models studied from an algebraic viewpoint are either too general or too restrictive to be used in practice. In this talk we consider algebraic time-reversible models on phylogenetic trees and introduce a new inner product to make all transition matrices of the process diagonalizable through the same orthogonal eigenbasis. This framework generalizes the Fourier transform widely used to work with group-based models. As illustration, we provide relevant phylogenetic invariants for trees evolving under the Tamura-Nei model of nucleotide substitution.

The Geometry of Colored Gaussian DAG Models

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Abstract: A colored directed acyclic graph can be used to represent the gaussian DAG models that fulfill a collection of partial homoscedasticity and partial homogeneity constraints placed on the model error variances and regression coefficients according to a coloring of the vertices and edges of the graph. We observe that the ideal generated by the conditional independence constraints imposed by the graph together with a set of polynomial constraints corresponding to the coloring is equal to the vanishing ideal of the model following saturation by the leading principal minors of the covariance matrix. This observation leads to a global Markov property with applications in causal discovery. We also observe that these models are smooth, making the models amenable to likelihood ratio tests.

The Voronoi diagram of codimension one varieties under polyhedral norms

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Abstract: Given a point x in a real algebraic variety X, which points in space are closer to x than to any other point in X? The answer to this question is precisely the so called *Voronoi cell* of x, and the set of all Voronoi cells of points in the variety is called the *Voronoi diagram* of X. Describing the Voronoi diagram of a variety is a fundamental problem in Metric Algebraic Geometry, and it has been extensively studied for the Euclidean distance. In this talk we explore the description of the Voronoi diagram of algebraic varieties of codimension one when the distance arises from a polyhedral norm. We will exemplify these diagrams with varieties arising from Algebraic Statistics and optimal transport.

The Wonderful Geometry of the Vandermonde Map

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Abstract: The Vandermonde map is the polynomial map given by the power-sum polynomials. We study the geometry of the image of the nonnegative orthant under under this map and focus on the limit as the number of variables approaches infinity. We will show, the geometry of this limit is the key to new undecidability results in nonnegativity of symmetric polynomials and deciding validity of trace inequalities in linear algebra.

On properties of the signed reduced A-discriminant

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Abstract: Computing the isotopy type of a hypersurface, defined as the positive real zero set of a polynomial, is a challenging problem in real algebraic geometry. For a fixed signed support, that is, for fixed exponent vectors and signs of the coefficients of the polynomial, the signed reduced A-discriminant divides the coefficient space into chambers such that in each chamber the isotopy type of the hypersurfaces is constant. Within unbounded chambers the isotopy type of the hypersurface can be described by a polyhedral object. However, hypersurfaces associated with bounded chambers might have isotopy types, which are more difficult to describe. In this talk, I will discuss properties of the signed support that preclude the existence of such bounded chambers of the signed reduced A-discriminant.

Gluing and splitting of homogeneous toric ideals

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Abstract: We show that two homogeneous affine semigroups can always be glued by embedding them suitably in a higher dimensional space. As a consequence, we will see that the sum of their toric homogeneous ideals is again a homogeneous toric ideal. We will apply our results to answer some questions on the splitting of toric ideals. In particular, we will see that two toric ideals associated to graphs always occur as the splitting of a toric ideal associated to a graph or an hypergraph whose minimal graded free resolution is the tensor product of the resolutions of the two original ideals. This is a sequel of a series of papers by the same authors. Some results recently obtained in [G. Favacchio, J. Hofscheier, G. Keiper, A. Van Tuyl: J. Algebra 574 (2021), 409–433] are recovered and improved.

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Combinatorial study of morsifications of real univariate singularities

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Abstract: We study a broad class of morsifications of germs of univariate real analytic functions. We characterize the combinatorial types of the resulting Morse functions, via planar contact trees constructed from Newton-Puiseux roots of the polar curves of the morsifications.

On the eigenvalues of Q-matrices and P-matrices

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Abstract: P-matrices are matrices all of whose principal minors are positive. Q-matrices are matrices whose sums of principal minors of the same order are positive. A matrix is a QM-matrix if all its powers are Q-matrices. The study of the eigenvalues of these matrices (and its powers) brings many open questions: for example, it is not know if the eigenvalues of a matrix A such that A and A^2 are P-matrices necessarily have positive real parts. In order to get a complete answer to these open questions we will characterise the real QM-matrices up-to size 5 and we characterise those real matrices A, 4×4, such that A and A^2 are Q-matrices but not all eigenvalues of A have positive real part.

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Duality in mass-action networks: A step closer to a new case of the Global Attractor Conjecture?

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Abstract: Mass-action networks can be regarded as polynomial ODEs, making their steady states semi-algebraic varieties. Within this framework, we introduce maximal invariant polyhedral supports and we prove that the set of preclusters is dual to the set of maximal invariant polyhedral supports. Precusters are special cases of the clusters introduced in 2012 by Conradi and Flockerzi. Given the close relation between maximal invariant polyhedral supports and siphons, we conjecture that there is a duality relation between siphons and clusters, which, we believe, based on the recent work of Craciun et al., might lead to the absence of boundary steady states in toric dynamical systems with small codimensional invariant polyhedra.

Measure-preserving mappings from the unit cube to spheres and projective spaces

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Abstract: The problem of finding measure-preserving mappings from one manifold to another has received considerable attention over the past few years due to their numerous applications in fields like computer graphics, medical imaging, signal processing, or, in general, in any area that requires good discretizations of a certain space. Thus, when looking for uniform collections of points or uniform grids (that is, grids all of whose cells have the same volume) on a manifold $\mathcal{M}$, a frequent approach consists in generating points or grids with that property on a simpler, easily discretizable space, like the unit cube, and then transporting them to $\mathcal{M}$ through a measure-preserving mapping. In this sense, most of the research has been carried out for two-dimensional and three-dimensional manifolds, but little is known for higher-dimensional spaces.
In this talk, I will show how to construct measure-preserving mappings from the unit d-dimensional cube to the compact rank one symmetric spaces, namely the d-dimensional sphere, the real, complex, and quaternionic projective spaces, and the Cayley plane.

Importance measures of multi-state systems via monomial ideals

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Abstract: In this talk we will introduce multi-state systems and how they are related to monomial ideals. Then, we will explain what importance measures are and how we can use algebra, in particular, a tool called support posets, to identify the most important structural components of a multi-state system.// En esta charla introduciremos los sistemas multi-estado y la relación existente entre ellos y los ideales lonomiales. Después, explicaremos qué son las medidas de importancia y cómo podemos utilizar el álgebra, en particular una herramienta llamada support poset, para identificar las componentes más importantes estructuralmente de un sistema multi-estado.