New Milne-type inequalities via fractional calculus

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Abstract: Inequalities play a main role in pure and applied mathematics.
In particular, the following Milne inequality plays an important role in the study of Rosseland's integral for the stellar absorption, see [1].

Proposition. *Let $\phi : (0,\infty) \to [0,\infty)$ be a Riemann integrable function with $\int_0^\infty \phi(x)\,dx = 1$. Let $a_i > 0$ and $f_i : (0,\infty) \to (0,\infty)$ such that ϕ/f_i is a Riemann integrable function on $(0,\infty)$ for $1 \le i \le n$. Then,*

$$\frac{1}{\int_0^\infty \frac{\phi(x)\,dx}{a_1 f_1(x)+\cdots+a_n f_n(x)}} \ge \frac{a_1}{\int_0^\infty \frac{\phi(x)\,dx}{f_1(x)}} + \cdots + \frac{a_n}{\int_0^\infty \frac{\phi(x)\,dx}{f_n(x)}} .$$

In this work we prove the following generalization of Milne inequality:

Theorem. *Let μ be a measure on the space X, $a_n \ge 0$ and $f_n : X \to [0,\infty]$ measurable functions for $n \ge 1$. Then,*

$$\frac{1}{\int_X \frac{d\mu(x)}{\sum_{n=1}^\infty a_n f_n(x)}} \ge \sum_{n=1}^\infty \frac{a_n}{\int_X \frac{d\mu(x)}{f_n(x)}} .$$

The proof of this inequality appears in the paper [2].

References

[1] E. A. Milne (1925). Note on Rosseland's integral for the stellar absorption. Monthly Notic. Royal Astron. Soc., 85, 979-984.

[2] J. M. Rodríguez, J. M. Sigarreta, E. Tourís (2023). New Milne-type inequalities via fractional calculus. Submitted.

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