Representability of orderings on the real line: a survey.

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Abstract: Can any qualitative scale be converted into an (equivalent) quantitative one? We will introduce here a survey of results concerning the representability of different kind of orderings (qualitative scales, preferences, etc.) into quantitative or numerical ones. From the point of view of Real Analysis, this will correspond to a deep understanding of the axioms of the real line that are related to order. For instance, we may wonder which totally ordered sets $(X, \preceq)$ are order-isomorphic to a subset of $\mathbb{R}$ (endowed with the usual order $\leq$). Apart from total orders, we will pay attention to some other classical ordered structures as complete preorders, interval orders and semiorders, commenting the key results about their numerical representability, that have been obtained in recent past years. Miscellaneous applications (e.g.. utility functions in Economics) will also been discussed.

References

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