Dynamics with zero Lyapunov exponents: From matrix cocycles to partial hyperbolicity

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Abstract: Beyond the well-studied class of uniformly hyperbolic dynamics, zero Lyapunov exponents may be unremovable and quite ubiquitous. Studying the lack of hyperbolicity (zero exponents) is difficult due to the absence of tools such as the existence of invariant manifolds provided by the Pesin theory. Ironically, there are settings where the nonhyperbolic part of dynamics can be described borrowing information from the hyperbolic one (non-zero exponents). I will explain how this can be done adopting a geometrical approach to coarsely describe different hyperbolic regimes and transitions between them. Applications include random products of 2×2 matrix cocycles and extend to robustly transitive nonhyperbolic dynamics. A crucial part is a description of topological properties of the space of measures in the spirit of the approximation results by Sigmund (in hyperbolic settings).

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