

# Forcing axioms and the size of the continuum

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**Abstract:** Georg Cantor famously proved in the 1870's that there are more real numbers than natural numbers. A natural question is then "Exactly how many reals are there?  $\aleph_1$ ?  $\aleph_2$ ? Maybe more?" This is known as the Continuum Problem. This problem has been one of the most important guiding ideas throughout the history of set theory. By work of Kurt Gödel in the 1930's and of Paul Cohen in the 1960's, we know that the standard axiomatic system for set theory, namely ZFC, does not solve the Continuum Problem. On the other hand, and notwithstanding the independence results of Gödel and Cohen, there are good reasons not to take the Continuum Problem as a pseudo-problem.

In my talk I will start by hinting at the reasons not to take the independence results as the last word in this story. Building on the framework I will lay out, I will then introduce and motivate forcing axioms and will present some old, and recent, results using these axioms indicating that the Continuum Problem may have a precise answer after all. I will also mention some competing views and open questions in the area.